# Corona Discharge Simulation of Multiconductor Electrostatic Precipitators

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**The paper considers corona discharge problems with multiple conductors, such as those appearing in some electrostatic precipitators. A common, precise condition is identified in which previous approaches proposed in literature fail. For dealing with these conditions, a novel formulation of the problem is proposed. Moreover, a Newton-Raphson scheme is defined for iteratively solving a non-standard Petrov-Galerkin Finite Element discretization of the problem. The presented approach is validated on a benchmark for which an analytical solution is known.**

***Index Terms*—Corona Discharge, Electrostatic Precipitators, Coupled Problems**

## I. Introduction

Corona discharge, such as the one appearing in electrostatic precipitators, is commonly modeled by coupling electrostatic and electrokinetic problems [1], [2]. Such problems are mathematically formulated by a coupled Poisson's problem in the electrical potential variable and an advection problem in the electric charge density variable. The numerical solution of such problems is difficult, because of the strong coupling of the problems in part due to the assumed boundary condition for the advection problem, known as Peek's boundary condition [2]. In order to overcome these difficulties various approaches have been proposed in literature [2]. In this paper the general case of multiconductor electrostatic precipitators is considered. In this case firstly it is shown that the formulation of the coupled electrostatic and electrokinetic problems, proposed in literature and commonly adopted in practice, does not guarantee the existence of a unique solution. This problem corresponds to the fact that the numerical approaches reported in literature discretizing the coupled Poisson's and advection problems fail exactly in this situation. Secondly, a proper boundary condition for the advection problem is defined in such a way to solve the uniqueness problem. For this modified problem a numerical approach based on a nonstandard Petrov-Galerkin Finite Element Method discretization is proposed.

## II. Problem Formulation

Corona discharge in multiconductor electrostatic precipitators is usually modeled by coupling the electrostatic problem to an electrokinectic one in a region $\Omega$. The electrostatic problem is formulated as

$$\nabla \cdot (-\varepsilon \nabla \varphi) = \rho \tag{1}$$

in which $\varphi$ is the electric potential, $\varepsilon$ is the permittivity and $\rho$ is the electric charge density. Boundary conditions for this problem are usually of mixed Dirichlet and Neumann type, in the form

$$\varphi = \varphi_s \text{ in } \Sigma_\varphi \tag{2}$$

$$-\varepsilon \frac{\partial \varphi}{\partial n} = 0 \text{ in } \Sigma_d \tag{3}$$

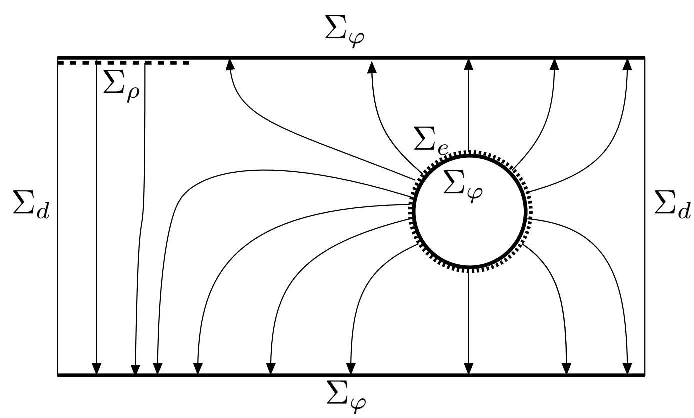


Fig. 1. Multiconductor electrostatic precipitator geometry.

in which $\Sigma_\varphi$ and $\Sigma_d$ are disjoint surfaces, the union of which is the complete boundary of the problem $\partial\Omega$. The electrokinetic problem is formulated as

$$\nabla \cdot (-\mu \rho \nabla \varphi) = 0 \tag{4}$$

in which $\mu$ is the charge mobility. (1) and (4) are coupled due to the presence of $\varphi$ and $\rho$ in both equations, and (4) is nonlinear due to the product of $\nabla\varphi$ with $\rho$. Boundary condition for this problem is Peek's condition, i.e. setting the electric field normal to the emitter surface $\Sigma_e$ according to Peek's formula

$$-\frac{\partial \varphi}{\partial n} = e_p \text{ in } \Sigma_e \tag{5}$$

in which the normal vector is considered pointing outward with respect to the emitter surfaces, and $e_p$ is Peek's value.

It is well known that an advection problem requires one boundary condition on each characteristic line. However, while in two-conductor electrostatic precipitators, Peek's condition on the emitter surface exactly guarantees such condition, in multiconductor electrostatic precipitators this, in general, does not hold. For instance, in Fig. 1, it is evident that along the characteristic lines starting from the surface $\Sigma_\rho$, on which $-\nabla\varphi \cdot \mathbf{n} > 0$, no condition is set in this way. This problem is straighforwardly solved by introducing the boundary condition

$$\rho = \rho_s \text{ in } \Sigma_\rho, \tag{6}$$

in which $\rho_s$ can be chosen as 0, since no charge is injected at boundary surfaces different from the emitter. In this way, surfaces $\Sigma_e$ and $\Sigma_\rho$ are disjoint surfaces, the union of which is the set of all boundary points at which $-\nabla\varphi \cdot \mathbf{n} > 0$. By imposing (5), (6), one condition is correctly set for each characteristic line of the advection problem.

## III. Numerical Method

In some electrostatic precipitators problems [2], boundary condition (5) is not imposed directly and instead boundary condition (6) is used also on $\Sigma_e$. In this case an iterative algorithm is usually adopted in which the coupled electrostatic and electrokinetic problems are separately solved until convergence. This approach is not always satisfactory due to the well-known convergence issues of simple iteration schemes. If on the other hand Peek's boundary condition is to be imposed directly other issues arise since (5) introduces additional coupling between the electrostatic and electrokinetic problems. All the above considerations lead to the following weak formulation for the electrostatic problem

$$\int_\Omega \nabla\varphi' \cdot \varepsilon \nabla\varphi + \int_{\Sigma_e} \varphi' \varepsilon e_p = \int_\Omega \varphi' \rho \tag{7}$$
$$\forall \varphi' | \varphi' = 0 \text{ in } \Sigma_\varphi \setminus \Sigma_e,\ \varphi = \varphi_s \text{ in } \Sigma_\varphi$$

This formulation is nonstandard since the test functions are not required to vanish on the whole Dirichlet boundary for $\varphi$ but only where Peek's condition is not set. The weak formulation for the electrokinetic problem reads

$$\int_\Omega \nabla\rho' \cdot \mu\nabla\varphi\rho + \int_\Omega \nabla\rho' \cdot \omega\nabla\rho + \int_{\Sigma_\varphi \setminus (\Sigma_e \cup \Sigma_\rho)} -\rho' \mu \frac{\partial\varphi}{\partial n}\rho = 0 \tag{8}$$
$$\forall \rho' | \rho' = 0 \text{ in } \Sigma_e \cup \Sigma_\rho,\ \rho = \rho_s \text{ in } \Sigma_\rho$$

Also this formulation is nonstandard since the test functions are required to vanish not only on the Dirichlet boundary for $\rho$ but also where Peek's condition has been set. Furthermore, with respect to (4) a stabilization term containing a parameter $\omega$ has been added.

A Newton-Raphson approach can be used to solve the coupled system formed by (7) and (8), thus at the $k$-th iteration estimates $\varphi^k$, $\rho^k$ are assumed, and more accurate estimates $\varphi^{k+1} = \varphi^k + \delta\varphi$, $\rho^{k+1} = \rho^k + \delta\rho$ are determined by solving the linearized coupled system given by

$$\begin{aligned} &\int_\Omega \nabla\varphi' \cdot \varepsilon\nabla\delta\varphi - \int_\Omega \varphi'\delta\rho = \\ &- \int_\Omega \nabla\varphi' \cdot \varepsilon\nabla\varphi^k + \int_\Omega \varphi'\rho^k - \int_{\Sigma_e} \varphi'\varepsilon e_p \\ &\forall\varphi'|\varphi' = 0 \text{ in } \Sigma_\varphi \setminus \Sigma_e,\ \delta\varphi = \varphi_s - \varphi^k \text{ in } \Sigma_\varphi \end{aligned} \tag{9}$$

and

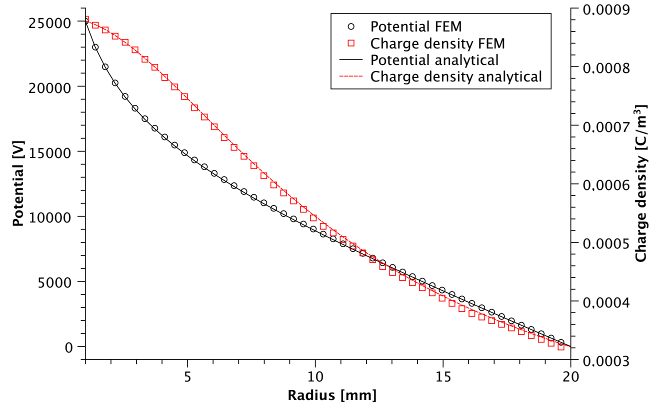


Fig. 2. FEM vs. analytical solution on benchmark problem

$$\begin{aligned} &\int_\Omega \nabla\rho' \cdot \mu\rho^k\nabla\delta\varphi + \int_\Omega \nabla\rho' \cdot \mu\nabla\varphi^k\delta\rho + \int_\Omega \nabla\rho' \cdot \omega\nabla\delta\rho - \\ &\int_{\Sigma_\varphi \setminus (\Sigma_e \cup \Sigma_\rho)} \rho'\mu\rho^k \frac{\partial\delta\varphi}{\partial n} - \int_{\Sigma_\varphi \setminus (\Sigma_e \cup \Sigma_\rho)} \rho'\mu \frac{\partial\delta\varphi^k}{\partial n}\delta\rho = - \int_\Omega \nabla\rho' \cdot \mu\nabla\varphi^k\rho^k \\ &+ \int_\Omega \nabla\rho' \cdot \omega\nabla\rho^k + \int_{\Sigma_\varphi \setminus (\Sigma_e \cup \Sigma_\rho)} \rho'\mu\partial\varphi^k/\partial n\rho^k \\ &\forall\rho'|\rho' = \ \text{in } \Sigma_e \cup \Sigma_\rho,\ \rho = \rho_s \text{ in } \Sigma_\rho \end{aligned} \tag{10}$$

At each iteration the $\Sigma_\rho$ surface is approximated by the $\Sigma_\rho^k$ surface, consisting of all boundary points, distinct from $\Sigma_e$, such that $-\nabla\varphi^k \cdot \mathbf{n} < 0$. Fig. 2 shows the results obtained by the proposed approach on a coaxial cylindrical benchmark problem, described in [4], for which an analytical solution is available.

## IV. Conclusions

The paper proposes a novel formulation to address corona discharge problems with multiple conductors, such as those appearing in some electrostatic precipitators. The method can cope, through (6), with configurations in which previous approaches proposed in literature fail. From the numerical point of view, a Newton-Raphson scheme is introduced for iteratively solving a non-standard Petrov-Galerkin Finite Element discretization of the problem. The presented approach is validated on a benchmark problem for which an analytical solution is known. The extended version of the paper will provide further details regarding the formulation, the stabilization of the advection equation, the use of first-order and second-order elements and test problems of industrial complexity.

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